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Let $(h, k)$ be the tangent point of the ellipse with a side.
Then $a=n^2/k$, $\frac{1}{2}b=m^2/n$, or $k/h=2a/b=n/m$. Also $2ah+bk=hk$.
$\therefore k=\frac{1}{2}a$, $h=\frac{1}{4}b$. $\therefore b^2n^2+4a^2m^2=16m^2n^2$. And $bd=2am$.
$\therefore m=\dfrac{b}{2\sqrt{2}}$, $n=\dfrac{a}{\sqrt{2}}$; (eccentricity)$^2=\dfrac{4a^2-b^2}{4a^2}$.

II. Solution by A. H. HOLMES, Brunswick, Me.

Let $2a$=base of the isosceles triangle, and $b$ its perpendicular height.

Construct on $2a$ an equilateral triangle, and inscribe in it a semi-circle its diameter collinear with base $2a$. Then the radius of the semi-circle will be $\frac{a\sqrt{3}}{2}$ which is one-half the perpendicular of the equilateral triangle. Now consider this triangle to be projected into an isosceles triangle whose base will be, of course, the same as that of the equilateral triangle, but whose perpendicular height is $b$. The semi-circle inscribed in the equilateral triangle will be projected into the maximum semi-ellipse that can be inscribed in the isosceles triangle, and one of its semi-axes will have the same proportion to the perpendicular of the isosceles triangle that the radius of the semi-circle has to the perpendicular of the equilateral triangle.

$\therefore$ Eccentricity of ellipse$=\dfrac{\sqrt{(b^2-3a^2)}}{b}$ or $\dfrac{\sqrt{(3a^2-b^2)}}{a\sqrt{3}}$, accordingly as $\sqrt{(a^2+b^2)}$ is greater or less than $2a$. If $b$=one of the sides,

$$\varepsilon=\sqrt{\frac{b^2-4a}{b^2-a^2}}, \text{ or } \frac{\sqrt{(4a^2-b^2)}}{a\sqrt{3}}.$$

Also solved by Jacob Westlund.

## DIOPHANTINE ANALYSIS.

123. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Of two numbers $a_ib_ic_id_ie_i$ $(i=1, 2)$ it is given that their 10 digits $a_1$, ....., $e_2$ form a permutation of 0, 1, ....., 9, and that the sum of the two is $x3951$. Give an immediate evaluation of $x$; also list the possible pairs $a_1, a_2$; .....; $e_1, e_2$.

Solution by the PROPOSER.

Since the sum of the 10 digits is 45, $x+18$ must be a multiple of 9 by the rule of casting out of 9's. Hence $x=9$.

Next, on adding the third column there cannot be 1 to carry; otherwise $c_1+c_2$ or $c_1+c_2+1$ would be 19, and $c_1 \gtrless 9$, $c_2 \gtrless 9$. Hence

(1) $b_1+b_2=3$, $a_1+a_2=9$; or (2) $b_1+b_2=13$, $a_1+a_2=8$.

If $e_1+e_2=1$, the $b$'s are not 0, 3; nor 1, 2. Hence in this case,

$e_1, e_2=0, 1$; $b_1+b_2=13$; $a_1+a_2=8$; $d_1+d_2=5$, $c_1+c_2=9$;
or $d_1+d_2=15$, $c_1+c_2=8$.

Thus $a_1, a_2 = 2, 6$ or $3, 5$; $d_1 + d_2 = 15$, giving sets I, II below. Let next $e_1 + e_2 = 11$. Then $d_1 + d_2 = 4$, $c_1 + c_2 = 9$; or $d_1 + d_2 = 14$, $c_1 + c_2 = 8$. From these and (1) or (2), we get III.....XI as the only sets.

| | $a_1, a_2$ | $b_1, b_2$ | $c_1, c_2$ | $d_1, d_2$ | $e_1, e_2$ |
|---|---|---|---|---|---|
| I | 2, 6 | 4, 9 | 3, 5 | 7, 8 | 0, 1 |
| II | 3, 5 | 4, 9 | 2, 6 | 7, 8 | 0, 1 |
| III | 1, 8 | 0, 3 | 2, 6 | 5, 9 | 4, 7 |
| IV | 3, 6 | 1, 2 | 0, 8 | 5, 9 | 4, 7 |
| V | 4, 5 | 0, 3 | 1, 7 | 6, 8 | 2, 9 |
| VI | 0, 9 | 1, 2 | 3, 5 | 6, 8 | 4, 7 |
| VII | 1, 7 | 5, 8 | 3, 6 | 0, 4 | 2, 9 |
| VIII | 2, 6 | 5, 8 | 0, 9 | 1, 3 | 4, 7 |
| IX | 3, 5 | 6, 7 | 1, 8 | 0, 4 | 2, 9 |
| X | 0, 8 | 4, 9 | 2, 7 | 1, 3 | 5, 6 |
| XI | 0, 8 | 6, 7 | 4, 5 | 1, 3 | 2, 9 |

Sets VI, X, and XI may properly be excluded.

Also solved by G. B. M. Zerr.

## MISCELLANEOUS.

132. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Six officers of different grades (1, 2, 3, 4, 5, 6) from each of six branches the army ($a$, $b$, $c$, $d$, $e$, $f$) are to be arranged in a square so that each rank and each file shall have an officer of each grade and each branch. Can it be done? If not, prove it. The arrangement of five officers of each kind is easy.

Remark by L. E. DICKSON, The University of Chicago.

This problem, proposed in the February, 1903, number, is here repeated to call attention to the fact that no solution has yet been sent to the editors. If, instead of 6, we employ an odd number $n$, we obtain an immediate solution with $a_1 b_2 c_3 \ldots\ldots \nu_n$ as the first row, $a_1 a_n a_{n-1} \ldots\ldots a_3 a_2$ as the main diagonal, the scheme being completed by permuting $a$, $b$, $c$, ....., $\nu$ cyclically, and 1, 2, ....., $n$ cyclically. Thus for $n=3$ we obtain the (single, notation apart) possible solution:

$$\begin{matrix} a_1 & b_2 & c_3 \\ c_2 & a_3 & b_1 \\ b_3 & c_1 & a_2 . \end{matrix}$$

The problem is impossible for $n=2$. I proceed to show that there are exactly two distinct solutions for $n=4$. I first find the possible schemes for the *letters*.

By interchange of columns, we may bring the $a$'s into the main diagonal. Call the first row *abcd*. If $b$ is fourth in the second row, we interchange the third and fourth row, the third and fourth columns, and permute $c$, $d$, and get *abcd* as